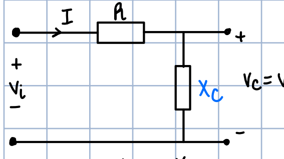
# Analogue




$H(j\omega) = \frac{V_0}{V_i} = \frac{X_C}{(R+X_C)} = \frac{1}{1+j\omega RC}$

freq. domain transfer function

Amplitude of trans. func.

$|H(j\omega)| = \left|\frac{1}{1+j\omega RC}\right| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_C}\right)^2}}$

Transfer Function

$H(j\omega) = |H(j\omega)| \angle \phi(j\omega)$

$-\arctan\left(\frac{\omega}{\omega_C}\right)$

$\omega_C = \frac{1}{RC}$

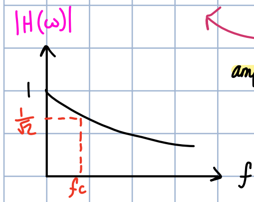
$f_C = \frac{\omega_C}{2\pi}$

corner (cut off or critical) angular ($\omega$) freq or freq. ($f$)

When $\omega = \omega_C$ or $f = f_C$

$|H(j\omega)| = \frac{1}{\sqrt{2}}$

→ Half power 3dB bandwidth




amplitude respone ↓ as freq ↑ ∴ low pass filter

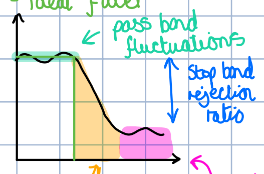
Freq. response of RC circuit

- Also get CR and LR circuits

## Filters continued...

Frequency response - usable frequency range

- ideal filter




- filter roll off with freq.

Producing Filter

Resistors respond equally to all frequencies

$V(f) = R \times I(f)$

Wave forms of V-I associated with resistor vary only in amplitude

Frequency dependant impedance

Capacitors

Stores energy in electric field (Farad)

Charge accumulate on plates as current passed into it

$q(t) = \int_{-\infty}^{t} i(t)\,dt$

$v(t) = \frac{q(t)}{C}$ → $\boxed{v(t) = \frac{1}{C}\int_{-\infty}^{T} i(t)\,dt}$ and $\boxed{i(t) = C\frac{dv(t)}{dt}}$

capacitance

Inductor (Henry)

Stores energy in magnetic field

→ induced magnetic field proportional to current flow

→ for sine waveform inductor results in similar but opposite equations.

$\boxed{v(t) = L\frac{di(t)}{dt}}$

'Storage' capability seperates phases of current and voltage by 90°

For sine waveform

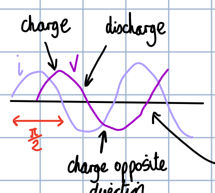
$v(t) = V_p \sin(\omega t)$

$i(t) = CV_p \frac{d[\sin(\omega t)]}{dt} = CV_p\omega\cos(\omega t)$

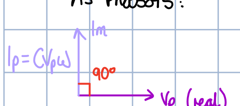
$i = CV_p\omega\sin(\omega t + \frac{\pi}{2})$

current is phase-shifted sine wave 90°

Amplitude ∝ ω and C




As Phasors:




$\vec{V_p} = V_p \angle 0$ or $V_p + j0$

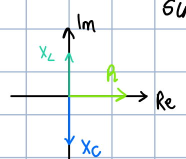
$\vec{I_p} = CV_p\omega \angle \frac{\pi}{2}$ or $\vec{I_p} = 0 + jCV_p\omega$

## Capacitive Reactance

- for capacitor, impedance is purely imaginary : reactance negative and decreases with frequency

$X_C = \frac{\vec{V}}{\vec{I}} = \frac{V_p}{jV_p\omega C} = -j\frac{1}{2\pi fC}$

capacitive reactance




$X_L = j2\pi fL$

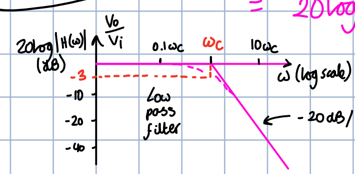
inductive reactance

purely imaginary positive increases with freq.

## Transfer function continued... (top right)

$|H(\omega)|$ in dB

$= 20\log_{10}\frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_C}\right)^2}}$




$\omega << \omega_C$ $|H(\omega)| = 1$ or 0dB

$\omega = \omega_C$ $= 1/\sqrt{2}$ $= -3$dB

$\omega >> \omega_C = -20\log\frac{\omega}{\omega_C}$

→ high freq amplitude decreases at rate of 20dB for f ↑ of 10x (20 dB per decade roll off)

## Sine wave

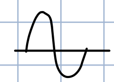
periodic : sine waves added equal spaced in frequency

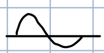
- lowest freq = freq of signal
- all others integer multiples of lower freq. → Harmonics

any waveform can be built adding sine waves

→ unperiodic → different freqs, A's and Ø's

Sine wave → Linear System → Sine output





same freq. different amplitude and phase

## Bandwidth

→ difference between half power frequencies

→ -3dB or $V = \frac{V_{max}}{\sqrt{2}}$

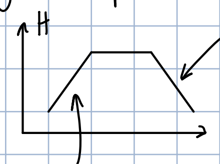
frequency domain → frequency spectrum Amplitude - phase graph

## Coscading Filters

Steeper transition bands

- Can create band-pass and band-stop filters

e.g band-pass




high + low pass coscaded

Output impedance of 1st and input of 2nd change respective freq. response

→ use buffer between

→ elims. loading effect